ON ASPECTS OF VISCOUS DAMPING FOR AN AXIALLY TRANSPORTING STRING

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ABSTRACT: *This research paper studies the damped linear homogeneous string-like equation. The two ends of the string are being held fixed and, the general initial conditions are considered. In physical viewpoint; this research problem describes the damped vertical vibrations of a band-saw blade or a chain drive. The second order partial differential equation for axially moving continuum has been formulated from energy principle such as Hamilton's principle. The axial velocity of the string is assumed to be positive, constant and small compared to wave velocity, and it is also assumed that the introduced viscous damping generates small damping in the system. To solve the mathematical equation of motion, a two timescales perturbation method is used to find the approximate analytic solutions. It is shown that the introduced damping does in fact affect the solution responses, and reduces the vibration and unnecessary noise in the system. It is also shown that the damping generated in the system does not depend on the mode numbers n.*

Keywords: band-saw blade, viscous damping, perturbation, oscillations

INTRODUCTION

Physical, mechanical and structural systems are arising in the class of vibratory systems. Axially translating elastic systems are one of them. These systems always experience the energy loss often known as energy dissipation, Refs. [1-5]. There are many examples of these axially translating systems, for example, see Refs [4,5] for conveyor belts; Refs. [4,6-7] for elevator systems, and other such kind of systems subject to oscillations due to machine motion, wind, traffic etcetra.

In this technological and advanced world, analysis and study of axially translating systems under internal, material, viscous and boundary damping has received great importance in manufacture.

Many a time damping devices are used to suppress the unnecessary vibrations in these systems, Refs. [5-6,8]. The mechanical motion of axially translating systems is described wave-like or beam-like equations.

The vibrations are generated when an inertial element displaces from its rest position due to transfer of kinetic energy to system and restoring force is created which pulls the element back towards the equilibrium position. If the vibration becomes more intensive then it is harmful for the mechanical system, that is why, researchers are trying to reduce it by using different tactics,for instance, damping is used to control the vibrations through the boundary support. In some cases damping is used through spatial coordinate of the system, Refs. [1, 3-6]. The damping can also be used to suppress the oscillation amplitudes of plates, see Ref. [9]. Recently, authors in Ref. [10] have discussed aspects of the reflection properties with damping and found out interesting outcomes. In Ref. [11], tensioned beam describing elevator system is studied.

This research work studies the string-like equation with small viscous damping throughout the whole spatial coordinate where the boundary conditions are assumed to be fixed and general initial conditions have been taken into account. To solve the equations of motion, a two timescales perturbation method together with separation of variables and eigenfunction expansion method, see Refs. [12-18], has been used to approximate the analytic approximations of the solutions, where it is found that the vibratory energy of the oscillation amplitudes is dissipated by the constant rate. To this point it is the important result from the mathematical and mechanical point of view.

THE EQUATION OF MOTION AND THE ANALYTIC APPROXIMATIONS

The Derivation of Equation of Motion by Hamilton's Principle

In this section, the equation of motion related to axially translating elastic system with the suitable initial and the boundary conditions have been constructed by the application of the Hamilton's principle. Consider a band-saw blade which is moving with an axial velocity V between two supports (pulleys) that are a distance L apart. The transversal vibrations of the band saw blade can be modeled as a tensioned string equation. The mathematical model of a traveling tensioned string is based on the assumptions that X is the spatial coordinate, T is the time,

$U(X, T)$ models the mechanical displacement field, the pretension, P, is constant in the string. The transversal vibrations are assumed to be small, the effects of the gravity are neglected, the mass of the string per unit length, ρ, is constant, the axial velocity, V, of the string is positive and assumed to be constant, and that the damping coefficient, β, is constant.

To obtain the equations of motion, the Hamilton's principle has been used. The kinetic energy of the band-saw blade of length L is given by;

$$K.E. = \frac{1}{2}\rho\int_0^L \left(U_T + \overline{V}U_X\right)^2 dX, \qquad (1)$$

and, that the potential energy is given by;

$$P.E. = \frac{1}{2}P\int_0^L U_X^2 dX, \qquad (2)$$

and, the virtual work done by the damping force on the string is

$$\delta W = -\beta\int_0^L \left(U_T\left(L,T\right)+\bar{V}U_X\left(L,T\right)\right)\delta U\left(X,T\right)dX, \quad (3)$$

where $U(X, T)$ is the displacement in the vertical direction, ρ is the mass density of the band-saw blade, V is the constant band-saw blade velocity, P is the constant pretension in the blade, X is the position along the horizontal axis, T is the time, L is the distance between the two pulleys, and, is the damping coefficient. By substituting Eq. (1), Eq. (2) and Eq. (3) into Hamilton's principle,

$$\int_{T_1}^{T_2}\left(\delta K.E.-\delta P.E.+\delta W\right)dT = 0, \quad (4)$$

and applying the variational operation, it follows that

$$\delta\bar{A} = \int_{T_1}^{T_2}\int_0^L\left\{-\rho\left(U_{TT}+2\bar{V}U_{XT}+\bar{V}^2U_{XX}+\dot{\bar{V}}U_X\right)+PU_{XX}\right.$$
$$\left.-\beta\left(U_T+\bar{V}U_X\right)\right\}\delta U\left(X,T\right)dXdT$$
$$+\int_{T_1}^{T_2}\left\{\rho\bar{V}\left(U_T+VU_X\right)-PU_X\right\}\delta U\left(X,T\right)\Big|_0^L dT$$
$$+\int_0^L\rho\left(U_T+\bar{V}U_X\right)\delta U\left(X,T\right)\Big|_{T_1}^{T_2}dX = 0. \quad (5)$$

The equation which is contained in the first term of Eq. (5), is called the equation of motion and is given by

$$\rho D_1 - PD_2 + \beta D_3 = 0,\ T>0,\ 0<X<L$$
$$\text{where } D_1 = \left(U_{TT}+2\bar{V}U_{XT}+\dot{\bar{V}}U_X+\bar{V}^2U_{XX}\right), D_2 = U_{XX} \quad (6)$$
$$\text{and } D_3 = \left(U_T+\bar{V}U_X\right)$$

The remaining terms are equivalent to specifying the initial and the appropriate boundary conditions. Following BCs are assumed:

$$U\left(0,T\right)=U\left(L,T\right)=0,\ T\geq 0 \quad (7)$$

where the initial displacement and the initial velocity are:

$$U\left(X,0\right)=F\left(X\right),\ \text{and } U_T\left(X,0\right)=H\left(X\right), \quad (8)$$

where the lettered subscript denotes the partial differentiation. To put the equation into non-dimensional form the following dimensionless quantities are used:

$$u=\frac{U}{L}, x=\frac{X}{L}, t=\frac{cT}{L}, V^*=\frac{\bar{V}}{c},$$
$$f\left(x\right)=\frac{F\left(X\right)}{L}, h\left(x\right)=\frac{H\left(X\right)}{c}, \bar{\delta}=\frac{\beta L}{\rho c},$$

where $c=\sqrt{\frac{P}{\rho}}$ is the wave speed.

Thus, the Eq. (6) into dimensionless form

$$u_{tt}+2V^*u_{xt}+\dot{V}^*u_x+V^{*2}u_{xx}-u_{xx}$$
$$+\bar{\delta}\left(u_t+V^*u_x\right)=0 \quad (9)$$

The BCs are given as,

$$u\left(0,t\right)=u\left(1,t\right)=0,\quad t\geq 0 \quad (10)$$

and the ICs,

$$u\left(x,0\right)=f\left(x\right),\ \text{and, } u_t\left(x,0\right)=h\left(x\right). \quad (11)$$

ANALYTICAL APPROXIMATIONS

In this section, an approximation of the solution of the IBVP (9)-(11) has been constructed by using the two timescales perturbation method as mentioned in section 3 of this paper in detail. Following two assumptions are made to utilize the multiple timescales perturbation method: (i) it is assumed that the band-saw blade velocity $\bar{V}$ is small compared to wave velocity $c=\sqrt{\frac{P}{\rho}}$, and (ii) it is assumed that βL is small compared to ρc. Based on these assumptions, it is reasonable to write $V^*=\frac{\bar{V}}{c}=\mathrm{O}(\varepsilon)$, and $\bar{\delta}=\frac{\beta L}{\rho c}=\mathrm{O}(\varepsilon)$, that is, $V^*=\varepsilon V$, and $\bar{\delta}=\varepsilon\delta$. Based on these assumptions; Eqs. (9)-(11) can be written as follows

$$u_{tt}-u_{xx}=-2\varepsilon Vu_{xt}-\varepsilon^2V^2u_{xx}-\varepsilon\delta u_t-\varepsilon^2\delta Vu_x \quad (12)$$

the boundary conditions, $u\left(0,t;\varepsilon\right)=u\left(1,t;\varepsilon\right)=0,$ (13)

and the initial conditions,

$$u\left(x,0;\varepsilon\right)=f\left(x\right),\ u\left(x,0;\varepsilon\right)=h\left(x\right), \quad (14)$$

By using a two timescales perturbation method the function $u(x,t;\varepsilon)$ is supposed to be a function of spatial variable x, the fast timescale t_1 and the slow timescale t_2. For this reason,

$$u\left(x,t;\varepsilon\right)=v\left(x,t_1,t_2;\varepsilon\right) \quad (15)$$

By using Eq. (15), the following transformations are needed for the time derivatives:

$$u_t=v_{t_1}+\varepsilon v_{t_2}$$
$$u_{tt}=v_{t_1t_1}+2\varepsilon v_{t_1t_2}+\varepsilon^2v_{t_2t_2} \quad (16)$$

By substituting Eqs. (15)-(16) into Eqs. (12)-(14); the problem in v up to $\mathrm{O}(\varepsilon)$ is given as follows

$$v_{t_1t_1}-v_{xx}=-2\varepsilon v_{t_1t_2}-2\varepsilon V\delta v_{xt_1}-\varepsilon\delta v_{t_1}+O\left(\varepsilon^2\right),$$
$$v\left(0,t_1,t_2;\varepsilon\right)=v\left(1,t_1,t_2;\varepsilon\right)=0,$$
$$v\left(x,0,0;\varepsilon\right)=f\left(x\right),$$
$$v_{t_1}\left(x,0,0;\varepsilon\right)+\varepsilon v_{t_2}\left(x,0,0;\varepsilon\right)=h\left(x\right). \quad (17)$$

It is usually assumed that the function

$u(x,t;\varepsilon)=v(x,t_1,t_2;\varepsilon)$ can be approximated by the powers of ε in the asymptotic expansion, that is:

$$v(x,t_1,t_2;\varepsilon)=v_0(x,t_1,t_2)+\varepsilon v_1(x,t_1,t_2)+\varepsilon^2\cdots, \tag{18}$$

and that all the v_j's for j = 0, 1, 2, $\cdots$; are found in such a way that no secular terms arise. It is also assumed that the unknown functions v_j are $O(1)$. Now, by substitution of Eq. (18) into Eq. (17), and then equating the powers of ε^0 and ε^1, and neglecting the ε^2 and the higher powers of ε, it follows that, the $O(1)$-problem is given as:

$$\begin{aligned}&v_{0t_1t_1}-v_{0xx}=0,\\&v_0(0,t_1,t_2)=v_0(1,t_1,t_2)=0,\\&v_0(x,0,0)=f(x),\qquad v_{0t_1}(x,0,0)=h(x),\end{aligned} \tag{19}$$

and, that the $O(\varepsilon)-$ problem is given as:

$$\begin{aligned}&v_{1t_1t_1}-v_{1xx}=-2v_{0t_1t_2}-2Vv_{0xt_1}-\delta v_{0t_1},\\&v_1(0,t_1,t_2)=v_1(1,t_1,t_2)=0,\\&v_1(x,0,0)=0,\qquad v_{1t_1}(x,0,0)=-v_{0t_2}(x,0,0).\end{aligned} \tag{20}$$

It is seen that the O(1)-equation has solution only for the positive eigenvalues, $\lambda=(n\pi)^2$, see for instance Ref. [18], that is

$$v_0(x,t_1,t_2)=\sum_{n=1}^{\infty}\begin{pmatrix}A_{n0}(t_2)\cos(n\pi t_1)+\\B_{n0}(t_2)\sin(n\pi t_1)\end{pmatrix}\sin(n\pi x), \tag{21}$$

where A_{n0} and B_{n0} are known as Fourier coefficients. The values $A_{n0}(0)$ and $B_{n0}(0)$ can be obtained by using the orthogonality properties and the ICs. The space dependent solutions $\sin(n\pi x)$ for n = 1, 2, 3, $\cdots$, satisfy the following orthogonality properties,

$$\begin{aligned}\int_0^1\sin(n\pi x)\sin(m\pi x)\,dx&=0,\qquad \text{for}\qquad m\neq n\\&=\frac{1}{2},\qquad \text{for}\qquad m=n\end{aligned} \tag{22}$$

Thus, by using the initial conditions described in Eq. (19) and the orthogonality properties of the eigen-functions as given in Eq. (22), it follows that

$$A_{n0}(0)=2\int_0^1 f(x)\sin(n\pi x)\,dx, \tag{23}$$

$$n\pi B_{n0}(0)=2\int_0^1 h(x)\sin(n\pi x)\,dx. \tag{24}$$

Now, to solve the $O(\varepsilon)-$ problem, the eigenfunction expansion method is introduced, that is,

$$v_1(x,t_1,t_2)=\sum_{m=1}^{\infty}w_m(t_1,t_2)\sin(m\pi x), \tag{25}$$

where $w_m(t_1,t_2)$ are the unknown functions of t_1 and t_2. By substituting the Eq. (25) into the $O(\varepsilon)-$ equation, it follows that,

$$\begin{aligned}&\sum_{m=1}^{\infty}\left(w_{mt_1t_2}(t_1,t_2)+(n\pi)^2w_m(t_1,t_2)\right)\sin(m\pi x)=\\&-2v_{0t_1t_2}-2Vv_{0xt_1}-\delta v_{0t_1},\end{aligned} \tag{26}$$

By substituting $v(x,t_1,t_2)$ from Eq. (21) into Eq. (26), then by multiplying both sides with $\sin(n\pi x)$, then integrating the so-obtained equation from x = 0 to x = 1 with application of orthogonality conditions, it yields

$$\begin{aligned}&\omega_{nt_1t_1}+(n\pi)^2\omega_n=\\&\left(2n\pi A'_{n0}(t_2)+\delta n\pi A_{n0}(t_2)\right)\sin(n\pi t_1)\\&-\left(2n\pi \mathbf{B}'_{n0}(t_2)+\delta n\pi B_{n0}(t_2)\right)\cos(n\pi t_1)\\&+\sum_{m=1,m\neq n}^{\infty}\left\{4V(m\pi)^2\begin{pmatrix}A_{m0}(t_2)\sin(m\pi t_1)\\-B_{m0}(t_2)\cos(m\pi t_1)\end{pmatrix}\right\}\Theta_{mn}\end{aligned} \tag{27}$$

where Θ_{mn} are constants depending on the indices m and n. Their values are given by the following equation,

$$\Theta_{mn}=\int_0^1\cos(m\pi x)\sin(n\pi x)\,dx \tag{28}$$

To solve the Eq. (27), undetermined constants method has been applied. The solution of Eq. (27) consists homogeneous solution and the particular integral. If the first non-homogeneous term is considered, which is,

$$\begin{aligned}&\left(2n\pi A'_{n0}(t_2)+\delta n\pi A_{n0}(t_2)\right)\sin(n\pi t_1)\\&-\left(2n\pi B'_{n0}(t_2)+\delta n\pi B_{n0}(t_2)\right)\cos(n\pi t_1)\end{aligned}$$

then right assumption for the particular integral for Eq. (27) related to this non-homogeneous term will be

$$\begin{aligned}\omega_{np_1}(t_1,t_2)&=D(t_2)t_1\sin(n\pi t_1)\\&+E(t_2)t_1\cos(n\pi t_1)\end{aligned} \tag{29}$$

where $D(t_2)$ and $E(t_2)$ are unknown. Thus, by using the above assumption in Eq. (27), it yields

$$\begin{aligned}D(t_2)&=-\left(B'_{n0}(t_2)+\frac{\delta}{2}B_{n0}(t_2)\right),\\E(t_2)&=-\left(A'_{n0}(t_2)+\frac{\delta}{2}A_{n0}(t_2)\right).\end{aligned} \tag{30}$$

Now, using Eq. (30) into Eq. (29), it yields

$$\omega_{np_1}\left(t_1,t_2\right)=-\left(B'_{n0}\left(t_2\right)+\frac{\delta}{2}B_{n0}\left(t_2\right)\right)t_1\sin\left(n\pi t_1\right)$$
$$-\left(A'_{n0}\left(t_2\right)+\frac{\delta}{2}A_{n0}\left(t_2\right)\right)t_1\cos\left(n\pi t_1\right). \quad (31)$$

In Eq. (31) the solutions are unbounded in t_1. Such behavior of solutions is often known as the secular behavior. To have secular free behavior, following conditions are imposed in Eq. (27):

$$2A'_{n0}\left(t_2\right)+\delta A_{n0}\left(t_2\right)=0.$$
$$2B'_{n0}\left(t_2\right)+\delta B_{n0}\left(t_2\right)=0. \quad (32)$$

The above equations are uncoupled ODEs and they yield the solutions,

$$A_{n0}\left(t_2\right)=A_{n0}\left(0\right)e^{-\frac{\delta}{2}t_2},\quad \mathbf{B}_{n0}\left(t_2\right)=\mathbf{B}_{n0}\left(0\right)e^{-\frac{\delta}{2}t_2}, \quad (33)$$

where $A_{n0}(0)$ and $B_{n0}(0)$ are given in Eqs. (23) and (24). Thus, by using Eq. (33) into Eq. (21), the zeroth order approximation $v_0(x,t_1,t_2)$ can be written as,

$$v_0\left(x,t_1,t_2\right)=$$
$$\sum_{n=1}^{\infty}e^{-\frac{\delta}{2}t_2}\begin{pmatrix}A_{n0}\left(0\right)\cos\left(n\pi t_1\right)\\+B_{n0}\left(0\right)\sin\left(n\pi t_1\right)\end{pmatrix}\sin\left(n\pi x\right). \quad (34)$$

Now by substituting $t_2=\varepsilon t$ into the expression $-\frac{\delta}{2}t_2$, and then by dividing the so-obtained expression by t, it follows that the damping parameter Γ_n for all oscillations modes can be expressed by,

$$\Gamma_n=-\varepsilon\frac{\delta}{2}. \quad (35)$$

Thus, from Eq. (27) with Eq. (32), it follows that

$$\omega_{nt_1t_1}+\left(n\pi\right)^2\omega_n=$$
$$\sum_{m=1,m\neq n}^{\infty}\left\{4V\left(m\pi\right)^2\begin{pmatrix}A_{m0}\left(t_2\right)\sin\left(m\pi t_1\right)\\-\mathbf{B}_{m0}\left(t_2\right)\cos\left(m\pi t_1\right)\end{pmatrix}\right\}\Theta_{mn}. \quad (36)$$

Thus, the homogeneous solution to Eq. (36) is given by

$$\omega_{nH}\left(t_1,t_2\right)=A_{n1}\left(t_2\right)\cos\left(n\pi t_1\right)$$
$$+B_{n1}\left(t_2\right)\sin\left(n\pi t_1\right), \quad (37)$$

and the particular integral is followed as

$$\omega_{np}\left(t_1,t_2\right)=\sum_{m=1,m\neq n}^{\infty}\frac{4V\left(m\pi\right)^2\Theta_{mn}}{\left(n\pi\right)^2-\left(m\pi\right)^2}\begin{pmatrix}A_{m0}\left(t_2\right)\cos\left(m\pi t_1\right)\\-B_{m0}\left(t_2\right)\sin\left(m\pi t_1\right)\end{pmatrix}. \quad (38)$$

Hence, the total solution to Eq. (36) is the sum of the homogeneous solution and the particular integral, so from Eq. (37) and Eq. (38) it follows that $w_n(t_1,t_2)$ can be written as

$$\omega_n\left(t_1,t_2\right)=\omega_{nH}\left(t_1,t_2\right)+\omega_{nP}\left(t_1,t_2\right)$$
$$=A_{n1}\left(t_2\right)\cos\left(n\pi t_1\right)+\mathrm{B}_{n1}\left(t_2\right)\sin\left(n\pi t_1\right)+$$
$$\sum_{m=1,m\neq n}^{\infty}\frac{4V\left(m\pi\right)^2\Theta_{mn}}{\left(n\pi\right)^2-\left(m\pi\right)^2}\begin{pmatrix}A_{m0}\left(t_2\right)\cos\left(m\pi t_1\right)\\-B_{m0}\left(t_2\right)\sin\left(m\pi t_1\right)\end{pmatrix} \quad (39)$$

where $A_{n1}(t_2)$ and $B_{n1}(t_2)$ are still unknown functions. Thus, the Eq. (25) with the Eq. (39) can be expressed as (34)

$$v_1\left(x,t_1,t_2\right)=$$
$$\sum_{n=1}^{\infty}\left\{\begin{matrix}A_{n1}\left(t_2\right)\cos\left(n\pi t_1\right)+\mathrm{B}_{n1}\left(t_2\right)\sin\left(n\pi t_1\right)\\+\sum_{m=1,m\neq n}^{\infty}\frac{4V\left(m\pi\right)^2\Theta_{mn}}{\left(n\pi\right)^2-\left(m\pi\right)^2}\begin{pmatrix}A_{m0}\left(t_2\right)\cos\left(m\pi t_1\right)\\-B_{m0}\left(t_2\right)\sin\left(m\pi t_1\right)\end{pmatrix}\end{matrix}\right\}$$
$$\sin\left(n\pi x\right) \quad (40)$$

Now, by using inner product (22) and ICs (20) in Eq. (40), $A_{n1}(0)$ and $B_{n1}(0)$ are given by

$$A_{n1}\left(0\right)=-\sum_{m=1,m\neq n}^{\infty}\frac{4V\left(m\pi\right)^2\Theta_{mn}}{\left(n\pi\right)^2-\left(m\pi\right)^2}A_{m0}\left(0\right), \quad (41)$$

$$n\pi B_{n1}\left(0\right)=\sum_{m=1,m\neq n}^{\infty}\frac{4V\left(m\pi\right)^3\Theta_{mn}}{\left(n\pi\right)^2-\left(m\pi\right)^2}B_{m0}\left(0\right)$$
$$-A'_{n0}\left(0\right). \quad (42)$$

It is observed that the solution $v_1(x,t_1,t_2)$ still contains infinitely many undermined functions $A_{n1}(t_2)$ and $B_{n1}(t_2)$, for n = 1, 2, 3, $\cdots$. Since these unknowns functions can be used to prevent the unbounded terms in the solution of $v_2(x,t_1,t_2)$. It remains always reasonable to take $A_{n1}(t_2)=A_{n1}(0)$ and $B_{n1}(t_2)=B_{n1}(0)$. So far a formal approximation $v(x,t_1,t_2)=v_0(x,t_1,t_2)+\varepsilon v_1(x,t_1,t_2)$ has been constructed for $u(x,t)$, where $v_0(x,t_1,t_2)$ and $v_1(x,t_1,t_2)$ are continuously differentiable with respect to t_1, x, and t_2.

RESULTS AND DISCUSSIONS

In this section we are commenting, interpreting and explaining the results obtained in the previous Section 2. Using the complete analytical solution of the $O(1)$-problem, the influence of the small parameter $0<\varepsilon<<1$, and the damping parameter on the axially transporting system will be discussed in detail.

By using the $O(1)$- and the $O(\varepsilon)-$ solutions, it follows that

$$u(x,t)=\sum_{n=0}^{\infty} e^{-\frac{\delta}{2}\varepsilon t}\begin{pmatrix} A_{n0}(0)\cos(n\pi t) \\ +B_{n0}(0)\sin(n\pi t)\end{pmatrix}\sin(n\pi x), \tag{43}$$

where $A_{n0}(0)$ and $B_{n0}(0)$ are constants such as

$$A_{n0}(0)=2\int_0^1 f(x)\sin(n\pi x)\,dx\ , \tag{44}$$

$$n\pi B_{n0}(0)=2\int_0^1 h(x)\sin(n\pi x)\,dx\ , \tag{45}$$

.

Now from physical point of view all mathematical terms can be explained in the solution (43). In Eq. (43), the terms $\left(A_{n0}(0)\cos(n\pi t)+B_{n0}(0)\sin(n\pi t)\right)$ are oscillation terms obtained from time-dependent part of the IBVP. These terms oscillate with frequencies $n\pi$ for n = 1, 2, 3, $\cdots$. The term $e^{-\frac{\delta}{2}\varepsilon t}$ has arisen due to the viscous damping introduced in the system. This term indicates that as the time parameter t increases for fixed values of δ and ε the size of the oscillation amplitudes $A_{n0}(0)$ and $B_{n0}(0)$ will start to decrease. The last term $\sin(n\pi x)$ is the solution of the space-dependent part, which describes the shapes of the oscillation curves along x-axis ($0 < x < 1$) for fixed values of the time parameter t. By using Maple 16, two plots of the solution (43) are obtained for different values of the parameters ε and fixed δ , different number of modes n and for some different ranges of time intervals.

For the fixed values of $\varepsilon=0.01, \delta=1, n=5, x=0.5, t=0$ to $100,$ the solution in Eq. (43) has the following behavior

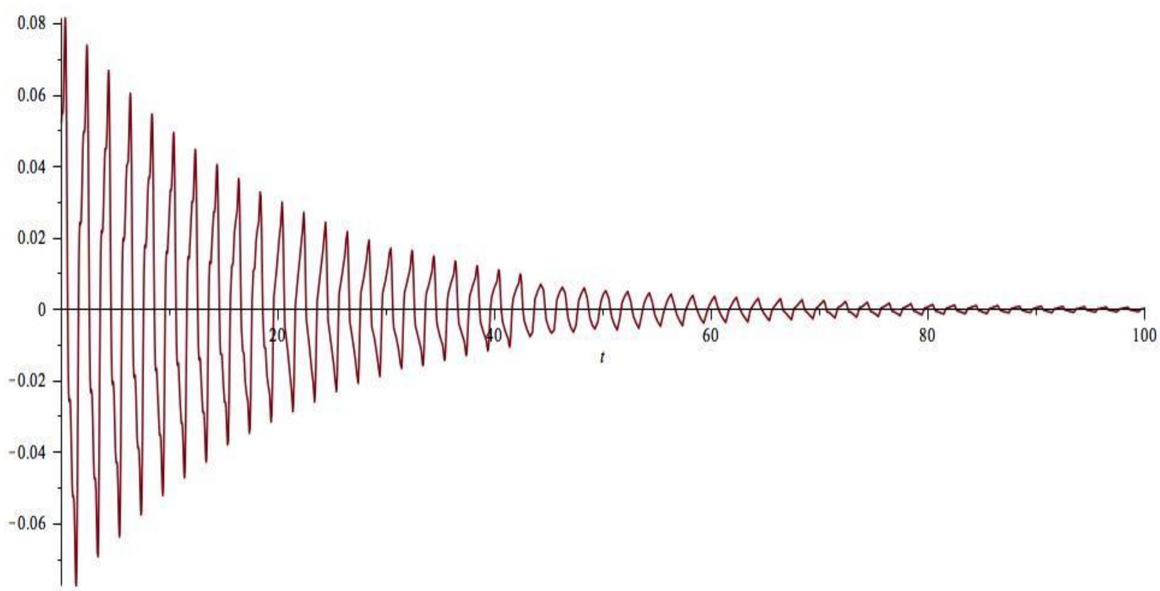

FIG. 1. A NUMERICAL PLOT OF EQ. (43)

For the fixed parameters: $\varepsilon=0.001, \delta=1, n=5, x=0.5, t=0$ to $200,$ the solution in Eq. (43) has the following behavior.

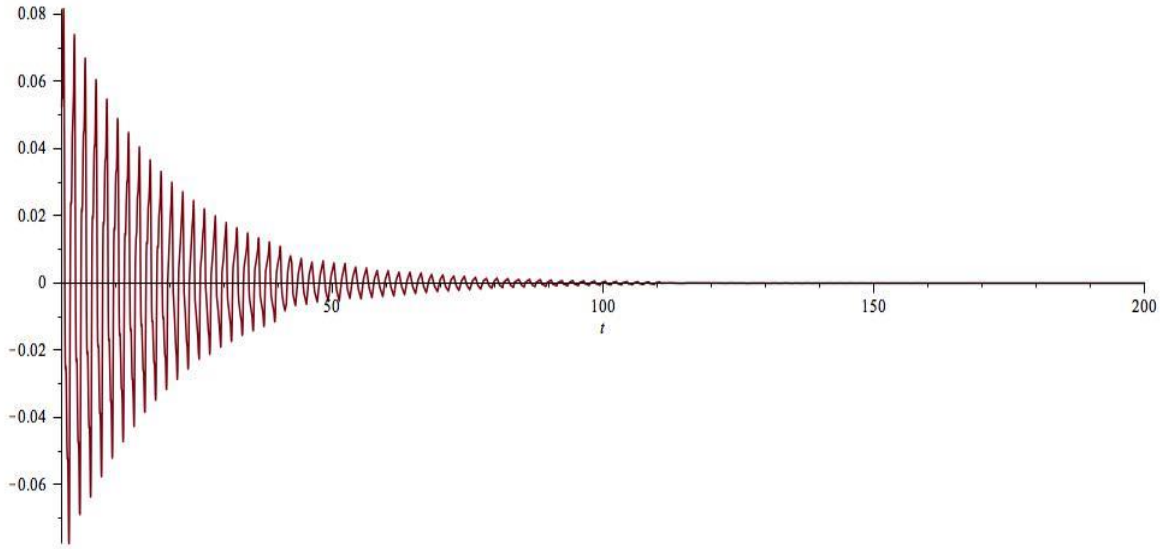

FIG. 2. A NUMERICAL PLOT OF EQ. (43).

In both of these plots, it can be seen that there is a decay in the oscillation amplitudes and this occurs due to fact that the damping has been introduced in the mechanical system. These graphs have very good agreement with the mechanical and physical expectations such that with increase in time the system gets into stable condition. From Fig. 1 and Fig. 2, it is clear that if the change occurs in the parameter ε there is huge effect on the decay rate of oscillation amplitudes. The decrease in ε results into slowing the damping effects in solutions.

CONCLUSIONS

By using a Hamilton's principle, a second order partial differential for axially translating continuous mechanical system under viscous damping has been constructed. A two timescales perturbation method is applied to solve an initial-boundary value problem to obtain closed form solutions of proposed mathematical model for the transversal vibrations of a transporting band-saw blade. It has been shown that all modes of oscillations are damped for the system and it has also been shown that the damping rates are completely independent of the mode number *n* and the physical system is being stable and vibratory energy dissipates.

REFERENCES

[1] Darmawijoyo and W.T. van Horssen, "On the Weakly Damped Vibrations of a String Attached to a Spring-Mass-Dashpot System", Journal of Vibration and Control, Volume 9, pp. 1231-1248, 2003.

[2] Darmawijoy and W.T. van Horssen, "On Boundary Damping for a Weakly Nonlinear Wave Equation", Springer, Nonlinear Dynamics, Volume 30, pp. 179-191, 2002.

[3] Darmawijoyo and W.T. van Horssen, "On a Rayleigh Wave Equation with Boundary Damping", Springer, Nonlinear Dynamics, Volume 33, No. 4, pp. 399-429, 2003.

[4] S.H. Sandilo and W.T. van Horssen, "On Boundary Damping for an Axially Moving Beam and On Variable Length Induced Oscillations of an Elevator Cable", In Proceedings of the 7th European Nonlinear Dynamics Conference (ENOC2011), 24-29 July, 2011, Rome, Italy.

[5] S.H. Sandilo and W.T. van Horssen, "On Boundary Damping for an Axially Moving Tensional Beam", ASME, Journal of Vibration and Acoustics, Volume 134, pp. 0110051-8, 2012.

[6] S.H. Sandilo and W.T. van Horssen, "On Aspects of Asymptotics for Axially Moving Continua", In PhD Thesis, Published by Delft University of Technology, The Netherlands, 2013.

[7] S.H. Sandilo and W.T. van Horssen, "On Small Variable Length Induced Oscillations of a Vertical String", In Book Problems of Nonlinear Dynamics and Condensed Matter Physics, published by Russian Academy of Sciences, Moscow, Russia, pp. 150-157, 2013.

[8] N. Gaiko and W.T. van Horssen, "On the Transverse, Low Frequency Vibrations of a Travelling String with Boundary Damping", ASME, Journal of Vibration and Acoustics, DOI: 10.1115/1.4029690, 2015.

[9] M.A. Zarubinskaya and W.T. van Horssen, "On Aspects of Boundary Damping for a Rectangular Plate", Journal of Sound and Vibration, Volume 292, No. 3-5, pp. 844-853, 2006.

[10] T. Akkaya and W.T. van Horssen, "Reflection and Damping Properties for Semi-Infinite String Equations with Non-Classical Boundary Conditions", Journal of Sound and Vibration, Volume 336, pp. 179-190, 2015.

[11] W.D. Zhu and J. Ni, "Energetics and Stability of Translating Media with an Arbitrary Varying Length", ASME, Journal of Vibration and Acoustics, Volume 122, pp. 295-304, 2000.

[13] A.H. Nayfeh, “Introduction to Perturbation Techniques”, WILEY-VCH Verlag GmbH and Co. KGaA, Weinheim, 2004.
[14] J. Kevorkian and J.D. Cole, “Multiple Scale and Singular Perturbation Methods”, Springer-Verlag, New York, 1996.
[15] A.H. Nayfeh and B. Balachandran, “Applied Nonlinear Dynamics”, WILEY-VCH Verlag GmbH and Co. KGaA, Weinheim, 2004.
[16] L. Meirovitch, “Principles and Techniques of Vibrations”, Prentice Hall, New Jersey, 1996.
[17] P. Hagedorn and A. DasGupta, “Vibrations and Waves in Continuous Mechanical Systems”, John Wiley and Sons, 2007.
[18] R. Haberman, “Applied Partial Differential Equations with Fourier Series and Boundary Value Problems”, Pearson, 5th edition, 2012.
[19] P. Hagedron and W. Seemann, “Modern Analytical Methods Applied to Mechanical Engineering System", In Modern Methods of Applied Mechanics and Their Applications”, edited by V.V. Ramyantev and A.V. Karapetyan, CISM Courses and Lectures no.387, Springer-Verlage, Wien New York, 317-328.